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Reply to the Comment by A. Rabhi on "Energies of the ground state and first excited 0^+ state in an exactly solvable pairing model"

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Abstract. It is pointed out that the "exact" solutions given in the recent Comment by A. Rabhi on the paper by N. Dinh Dang *Energies of the ground state and first excited* 0^+ *state in an exactly solvable pairing model* (Eur. Phys. J. A **16**, 181 (2003)) do not correspond to the standard exact solution of the well-known two-level pairing model. Other issues raised in the Comment reiterate the discussions already published in the original paper by N. Dinh Dang.

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In ref. [1] several approximations, namely the BCS approximation, Lipkin-Nogami method, random-phase approximation (RPA), quasiparticle RPA (QRPA), the renormalized RPA (RRPA), and renormalized QRPA (RQRPA), are tested by calculating the ground-state energy and the energy of the first excited 0^+ state using the well-known exactly solvable model with two symmetric levels interacting via a pairing force. The author of the recent Comment [2] discusses three points of ref. [1], namely i) the boson mapping in sect. 4.1.1, ii) the approximations leading to the RPA matrices (58)-(61) and those obtained in ref. [3], and iii) the approximation (75).

i) The author of [2] claimed that the one-boson energy $\omega^{(b)}_{\rm RPA}$ given by eq. (53) of [1] is erroneous. As a proof he introduces two "exact" solutions for the 0^+ energy in fig. 1 of [2]. However, these "exact" solutions are completely different from the standard exact solution of the well-known two-level model under consideration, which has been obtained using the $SU(2)$ algebra in many papers (see, *e.g.*, sect. 2 and fig. 3 of ref. [1], or refs. [4,5]). In particular, as has been mentioned at the end of sect. 4.1.1 of [1], the solution given by eq. (53) of [1] is exactly the same as that obtained for the first time by Högaasen-Fledman in ref. [4] using the space-variable technique. The author of the Comment [2], however, fails to reproduce the boson and exact solutions by Högaasen-Fledman, saying that he does not understand it[1].

ii) Section 4.1.2 just discusses two approximations, (62) and (63). The former is based on the exact commutation relations in eq. (10) and leads to the matrices (58)-(61). The latter leads to those in ref. [3]. The difference is the factor of 2 in the denominator of the second term at the right-hand side of eq. (58) in [1] and the corresponding term in eq. (38) of [3]. The omission of the q-term was considered in [1] also as a possible approximation, which leads to the appearance of the spurious mode, as has been pointed out in [1] and repeated by the author of [2]. It is worthwhile to study this approximation since there have been numerical calculations within the RPA neglecting the so-called scattering terms, which have the same origin as that of the q-term considered here (see the discussion in b) on page 185 of [1]). In such calculations the parameters of the effective interaction are usually readjusted in such a way that the energy of the spurious mode is zero to compensate for such effect (see, *e.g.*, ref. [6]). To my knowledge, there is no exact way to take into account the scattering term within the QRPA so far. Approximations to take into account the scattering term lead to the extended QRPA [7] or modified QRPA [8].

iii) The approximation (75) in [1] has been introduced so that one can compare the exact solution, the phonon solution obtained within the fermion formalism in sect. 4.1.2 with that obtained within the one-boson mapping in sect. 4.1.1. The conclusion is that the exact solution can be approximately considered as a mixture of

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[1] The derivation in eqs. (2)-(10) in [2] was actually sent by Dinh Dang to Rabhi after the latter failed to reproduce the result of eq. (53) of [1] as well as that obtained by Högaasen-Feldman in [4].

superfluid and normal-fluid states as developed in sect. 4.3. In this sense the approximation (75) mixes $N \pm 2$ states.

The RPA operator for the additional mode in the present two-level model is [9]

$$Q_{\rm a}^\dagger = X_2 A_2^\dagger - Y_1 A_1^\dagger \ . \tag{1}$$

The one for the removal mode is

$$Q_{\rm r}^\dagger = X_1 A_1^\dagger - Y_2 A_2^\dagger \ . \tag{2}$$

It is clear that neither $Q_{\rm a}^\dagger$ nor $Q_{\rm r}^\dagger$ conserves the particle number on each level because, according to the exact commutation relations in eq. (5) of [1], they do not commute with N_i $(i = 1, 2)$, except for $X_i = 0$ or $Y_i = 0$. So does not the operator $Q^\dagger = Q_{\rm a}^\dagger + Q_{\rm r}^\dagger$ in eq. (75) except for $X_i = Y_i = 0$. However, in the boson formalism, based on the mapping (49) and (50) of [1] with $\mathbf{b}_1 = \mathbf{b}_2 = \mathbf{b}$, as has been discussed in [2], one obtains the boson images of these commutation relations as follows:

$$[N_1, Q_{\rm a}^\dagger]_{\rm b} = [2\Omega - 2\mathbf{b}^\dagger\mathbf{b}, X_2\mathbf{b}^\dagger - Y_1\mathbf{b}] = -2X_2\mathbf{b}^\dagger - 2Y_1\mathbf{b} \ , \tag{3a}$$

$$[N_2, Q_{\rm a}^\dagger]_{\rm b} = [2\mathbf{b}^\dagger\mathbf{b}, X_2\mathbf{b}^\dagger - Y_1\mathbf{b}] = 2X_2\mathbf{b}^\dagger + 2Y_1\mathbf{b} \ , \tag{3b}$$

$$[N_1, Q_{\rm r}^\dagger]_{\rm b} = [2\Omega - 2\mathbf{b}^\dagger\mathbf{b}, X_1\mathbf{b} - Y_2\mathbf{b}^\dagger] = 2X_1\mathbf{b} + 2Y_2\mathbf{b}^\dagger \ , \tag{3c}$$

$$[N_2, Q_{\rm r}^\dagger]_{\rm b} = [2\mathbf{b}^\dagger\mathbf{b}, X_1\mathbf{b} - Y_2\mathbf{b}^\dagger] = -2X_1\mathbf{b} - 2Y_2\mathbf{b}^\dagger \ , \tag{3d}$$

Summing up eqs. (3a) and (3b), one finds

$$[N_1 + N_2, Q_{\rm a}^\dagger]_{\rm b} = [N, Q_{\rm a}^\dagger]_{\rm b} = 0 \ . \tag{4}$$

Summing up eqs. (3c) and (3d), one finds

$$[N_1 + N_2, Q_{\rm r}^\dagger]_{\rm b} = [N, Q_{\rm r}^\dagger]_{\rm b} = 0 \ . \tag{5}$$

Hence,

$$[N, Q^\dagger] = [N, Q_{\rm a}^\dagger + Q_{\rm r}] = 0 \ . \tag{6}$$

These results show that, in the boson formalism, although the additional and removal operators do not conserve the particle number N_i on each level, they do conserve the total particle number $N = N_1 + N_2$. So does the operator $Q^\dagger = Q_{\rm a}^\dagger + Q_{\rm r}^\dagger$ because of eq. (6).

In conclusion, the present Reply clarified several issues raised in the recent Comment [2] on paper [1] regarding the RPA and QRPA for an exactly solvable pairing model.

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